Floquet-Based FDTD Analysis of Two-dimensional Phased Array Antennas

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***Abstract*—The finite-difference time-domain method with Floquet boundary conditions has been used to calculate the radiation characteristics of one- and two-dimensional phased array antennas for different scan angles in E- and H-planes. Considerable savings in computer memory and computation time are realized since only the central 3 elements for a 1-D array and 3 × 3 elements for a 2-D array are to be modeled.**

I. Introduction

STUDY of the radiation characteristics of phased arrays of antenna elements that are periodic in one or two dimensions is an important problem in electromagnetics. Although solutions can be found for some simple radiating elements via modal analysis [1], most of the complex structures cannot be solved by this method. By assuming that a periodic structure is infinite in extent along one or both of the axes, the analysis can be simplified by the use of Floquet theorem [2]. In a time-domain algorithm such as the finite-difference time-domain (FDTD) method, this is tantamount to using a sequential time delay between adjacent elements of the antenna array. Although the use of the Floquet boundary conditions has previously been suggested in the literature [3], [4], to our knowledge neither details of a working algorithm nor its results have ever been reported. In this paper, first presented at the 1993 PIER Symposium [5], we describe the use of the FDTD method with one- and two-dimensional Floquet conditions to calculate the radiation characteristics of a phased array of rectangular apertures for various scan angles θ_o in E- and H-planes. We have recently become aware of a very similar approach being used by Tsay and Pozar [6], even though the method used by these authors has to date been limited to one-dimensional arrays and for broadside radiation.

To properly account for the coupling to leading as well as following elements, we have found it necessary to model a three-element arrangement for a periodic array in one dimension and a 3 × 3-element arrangement for a biperiodic array that is periodic along two axes. This allows a proper representation of both the time-delayed boundary for the latter element as well as the time-lead boundary for the earlier element. It is still a considerable improvement since only a limited number of radiating elements need to be modeled, and modeling of the more complex shapes of the radiating elements should therefore be possible with greatly reduced requirements of the computer memory. Even though the results are presented here for a 2-dimensional array of rectangular apertures, the method has also been used in our laboratory for a two-dimensional array of broadband flared notch radiators with rows of unit cells that are either parallel or staggered relative to one another in the plane of the array.

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II. Floquet-Based FDTD Method

Fig. 1 shows a 2-dimensional periodic array of identical radiating elements or unit cells. As indicated, the Floquet boundaries A, B, C, D are placed one unit cell removed from the central cell O on each of the four sides. For the principal lobe to be directed in θ_o, ϕ_o direction, the time delays τ_x, τ_y taken between the Floquet boundaries along x- and y-directions are $\tau_x = d_x sin\theta_o cos\phi_o/c$ and $\tau_y = d_y sin\theta_o cos\phi_o/c$, respectively. The absorbing boundaries with Higdon's boundary condition [7] are taken in xy planes in front of and behind the apertures. Fields in the time-delayed boundaries B and D can be easily obtained by saving the tangential electric or magnetic fields (E_t or H_t) at boundaries 2 and 4 in a time-delay buffer until they are needed for the time-delayed boundaries B and D. Consistent with the stability criterion for FDTD, the time step δt is taken to be $\delta_x/2c$ or $\delta_y/2c$, whichever is smaller. From Floquet boundary conditions, the fields at the preceding boundaries A and C in Fig. 1 need to be time advanced from the fields at boundaries 1 and 3 of the central unit cell. Since fields that occurred before time $t = 0$ at the start of the computer program cannot be known, a preliminary subprogram is run a number of time steps ahead of the main program in order to obtain these fields. The number of time steps used are $\tau_x/\delta t$ or $\tau_y/\delta t$, whichever is larger. Since we cannot know the time advanced fields for the preliminary subprogram, simple absorbing boundaries are used for boundaries A and C in the first instance.

Equations needed to satisfy Floquet boundary conditions are as follows:

For time-delayed boundaries B and D

$$\Psi_{tB}(x, y, z, t) = \Psi_{t2}(x, y, z, t - \tau_x)$$
$$\Psi_{tD}(x, y, z, t) = \Psi_{t4}(x, y, z, t - \tau_y) \quad (1)$$

For time-advanced boundaries A and C

$$\Psi_{tA}(x, y, z, t) = \Psi'_{t1}(x, y, z, t + \tau_x)$$
$$\Psi_{tC}(x, y, z, t) = \Psi'_{t3}(x, y, z, t + \tau_y) \quad (2)$$

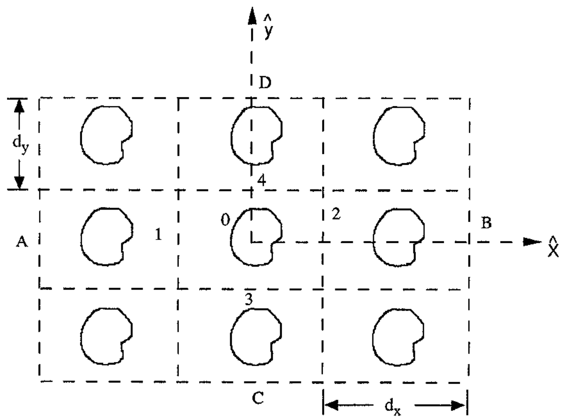

Fig. 1 A two-dimensional antenna array with the central unit cell O and Floquet boundaries A, B, C, and D.

where Ψ_t denotes E_t or H_t, whichever is important, and the second subscript denotes the various boundaries that are defined in Fig. 1. Ψ'_{t1} and Ψ'_{t3} are the tangential fields at boundaries 1 and 3 obtained from the preliminary subprogram.

For the results presented in this letter, we have used a 2-dimensional array of rectangular apertures with periodicities d_x = 2.54 cm (1"), d_y = 1.27 cm (0.5") along x- and y-axes. The aperture dimensions are 2.29 × 1.02 cm (0.9" × 0.4") along the respective axes. A Gaussian pulse of the form $E(t) = E_m e^{-\alpha[(n-n_{to})\delta t]^2}$ is prescribed as the excitation at the source plane for the purpose of obtaining a wideband response of the array where the parameter α may be altered to obtain the desired frequency spectrum of the pulse. For the calculations reported in this paper where the radiation characteristics are to be determined for the frequency band 7–10 GHz, we have used $\delta_x = \delta_y = \delta_z = 0.05$ for the Yee cells, $\delta t = 2.12 \times 10^{-12} s, \alpha = 16.7 \times 10^{20}, n_{to} = 42$ (selected for a lower initial value). Also, in order to reduce higher-order modes and concentrate on the TE_10 mode of excitation, we have prescribed the spatial variation $E_m(x)$ corresponding to the TE_10 mode and have taken a source plane at a depth of 20 cells into the waveguide. For Floquet boundary conditions (1) and (2), we have taken $\Psi_t = E_t$ for boundaries A and B and $\Psi_t = H_t$ for boundaries C and D to conform to the TE_{10} mode. We have used the E-fields calculated for a plane 5 cells in front of the source plane to obtain the reflection coefficient Γ of the aperture, and have used the calculated tangential E and H fields for the xy plane 1 cell above the aperture to calculate the far-field radiation pattern and the directivity of the unit cell.

III. Results

To check the accuracy of the procedure we have compared the calculated aperture field distributions of the central unit cell with those obtained for an "infinite" array modeled by 11 or more elements for a one-dimensional array along the x-axis. Shown in Fig. 2 are the calculated magnitude and phase distributions of E_y for the central unit cell for a scan angle θ_o = 60° in the H-plane. Nearly identical aperture distributions were also obtained using the Floquet-based FDTD method and the "infinite" array for all other scan angles $0° \leq \theta_o \leq 90°$.

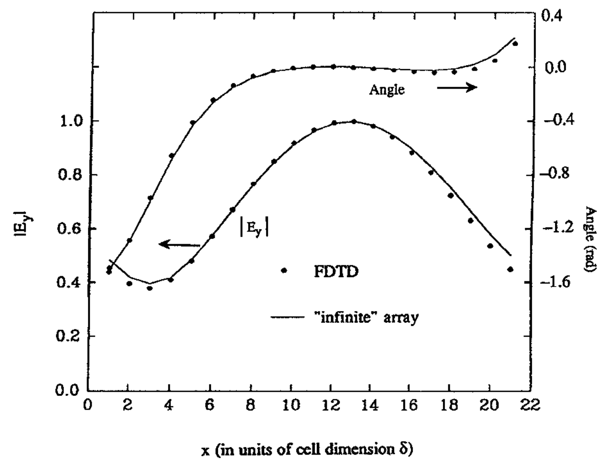

Fig. 2 The FDTD-calculated magnitude and phase distributions of E_y for the central rectangular element O at a plane 5 cells above the aperture. Also shown for comparison are the values calculated for an "infinite" array of 11 or more elements.

TABLE I
FDTD-Calculated Reflection Coefficients Γ for a 2-Dimensional Array of Rectangular Apertures

	H-plane		E-plane	
Angle	FDTD	Experimental Data	FDTD	Experimental Data
0°	0.153	0.156	0.153	0.156
10°	0.135	0.149	0.137	0.148
20°	0.085	0.131	0.131	0.126
30°	0.104	0.104	0.097	0.104
40°	0.095	0.073	0.096	0.077
50°	0.077	0.082	0.159	0.079
60°	0.066	0.174	0.309	0.173

We next used the method to calculate the directivities of the central element of the two-dimensional array of rectangular apertures as a function of the scan angle θ_o in both the H- and E-planes. Using the IBM 3090, the cpu time for each of the scan angles is about 12 minutes. The FDTD-calculated directivities at 10 GHz normalized relative to $D_o = 4\pi d_x d_y/\lambda^2 (= 4.51)$ are given in Fig. 3 as a function of θ_o. Also shown for comparison are the directivities obtained from the modal analysis [1] that are in excellent agreement with the FDTD-calculated results. The values of the reflection coefficients Γ of the central aperture for the various scan angles in H- and E-planes are given in Table I. Also given for comparison are the measured reflection coefficients for the corresponding conditions. The calculated results are in excellent agreement with the measured data for Γ for $0 \leq \theta_o \leq 50°$ for the H-plane and $0 \leq \theta_o \leq 40°$ for the E-plane.

IV. Conclusion

We have demonstrated the use of the finite-difference time-domain method in determining radiation characteristics of one- and two-dimensional phased array antennas for different scan angles in E- and H-planes. Use of the Floquet boundary conditions allows modeling of large antenna arrays by only 3 elements for a one-dimensional array and the central 3 × 3 elements for a two-dimensional array, resulting in substantial

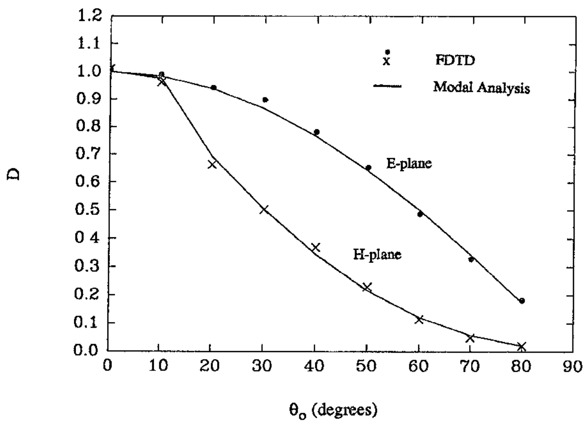

Fig. 3 Normalized directivity of the central radiating element of the 2-dimensional antenna array of rectangular elements at 10 GHz for scan angles θ_o in E- and H-planes, respectively. The directivity is normalized relative to $4\pi d_x d_y/\lambda^2 (= 4.51)$. Note the excellent agreement with the values obtained from the modal analysis [1].

savings of both the computer memory and computation time for numerical calculations. For a two-dimensional phased array of rectangular apertures, the directivities and reflection coefficients obtained for various scan angles in E- and H-planes are in excellent agreement with the results obtained from modal analysis [1] and from experimental data, respectively.

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